GAYATRI VIDYA PARISHAD COLLEGE OF ENGINEERING FOR WOMEN

(AUTONOMOUS)

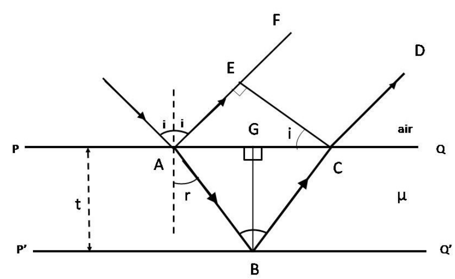
(Affiliated to Andhra University, Visakhapatnam)

I B.Tech. - I Semester Regular Examinations, December / January – 2025

ENGINEERING PHYSICS KEY

1. **a. Explain interference in thin films due to interference of reflected light and obtain the conditions for maxima and minima.**

Let us consider PQ and P'Q' are the two surfaces of a transparent film of uniform thickness t and refractive index μ as shown in figure. Let a ray of monochromatic light of wave length λ is incident on its upper surface at an angle i. This ray is partly reflected along AF and partly refracted along AB. After one reflection at B the ray moves in the direction of BC. After refraction at C, the ray finally emerges out along CD in air. As the rays AF and CD are derived from same source, therefore they are coherent. The find the path difference between the rays AF and CD, draw a normal CE on to AF.



The effective path difference is given by (AB+BC) in film – (AE) in air

$$\delta = \mu\,(AB+BC) - AE \qquad (1)$$

The angle ∠BGA and ∠BGC are equal to ∠r, therefore AB = BC, substituting this condition in equation (1)

The effective path difference $\delta = 2\mu.AB - AE$

From the right angled triangle ΔBGA, $\cos r = \frac{BG}{AB}$

$$\therefore AB = \frac{BG}{\cos r} = \frac{t}{\cos r}$$

From the right angled triangle ΔBGA, $\sin r = \frac{AG}{AB}$ $\therefore AG = AB \sin r$

From the right angled triangle ΔAEC, $\sin i = \frac{AE}{AC}$

$$\therefore AE = AC \sin i = 2AB \sin r \sin i$$

since AC= 2AG=2AB sin r

Therefore, AE = $\frac{2t}{\cos}$ sin i sin r $= \frac{2\mu t}{\cos r}$ $\sin^2 r$ (since sin i/ sin r = μ)

Thus the path difference between the two reflected rays :

$$\delta = 2.\frac{\mu t}{\cos} - 2.\frac{\mu t}{\cos r}.\sin^2 r$$

$$\delta = 2.\frac{\mu t}{\cos r}[1-\sin^2 r]$$

$$\delta = 2\mu t.\cos r$$

The above expression is known as "cosine law". It is clear from the above figure that the wave along AF reflected from denser medium, hence there occurs a phase change of $\boldsymbol{\pi}$ or path difference $\frac{\lambda}{2}$

∴ The additional path difference is $\frac{\lambda}{2}$

∴ The effective path difference $\delta = \mathbf{2\mu t \cos \boldsymbol{r}} + \frac{\lambda}{2}$

Condition for maxima and minima

For maxima the path difference should be equal to nλ

$$\therefore 2\mu t \cos r + \frac{\lambda}{2} = n\lambda \text{ or}$$

$$2\mu t \cos r = (2n-1)\frac{\lambda}{2} \qquad \text{where n= 1, 2, 3…}$$

For minima the path difference should be equal to $(2n+1)\frac{\lambda}{2}$

$$\therefore 2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2\mu t \cos r = n\lambda \qquad \text{where n= 0, 1, 2, 3…1.}$$

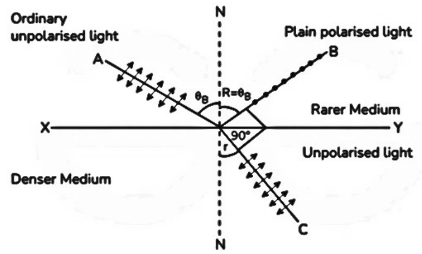
1. **b. What is Brewster's law? Show that reflected and refracted rays are perpendicular to each other at polarizing angle.**

Brewster's Law states that when unpolarized light strikes a surface at a specific angle (Brewster angle), the reflected light becomes polarized perpendicular to the plane of incidence. According to Brewster Law of polarization, the relative refractive index of a medium is equal to the tangent of the polarizing angle ($\boldsymbol{\theta_B}$).

$$\frac{\mu_2}{\mu_1} = tan\theta_B$$

Where μ_2 is the refractive index of the denser medium in figure and μ_1 is the same for rarer medium. If air is chosen for μ_1 then $\mu_1 = 1$

$$\mu_2 = tan\theta_B$$



$applying\ snell's\ law\ at\ point\ of\ incidence,$

$$\mu_1 sin\theta_B = \mu_2 \sin r$$

As $\mu_1 = 1$

$$sin\theta_B = \mu_2 \sin r$$

Using Brewster's law,

$$\mu_2 = tan\theta_B = \frac{sin\theta_B}{cos\theta_B} = \frac{\mu_2 \sin r}{cos\theta_B}$$

Therefore,

$\mu_2 = \frac{\mu_2 \sin r}{cos\theta_B}$ implies,

$$\sin r = cos\theta_B = \sin(90 - \theta_B)$$

Hence,

$$90 - \theta_B = r$$

$$90 = \theta_B + r$$

From geometry of figure

θ_B +r+(angle between reflected and refracted rays) =180°

from above derivation, it is proved that

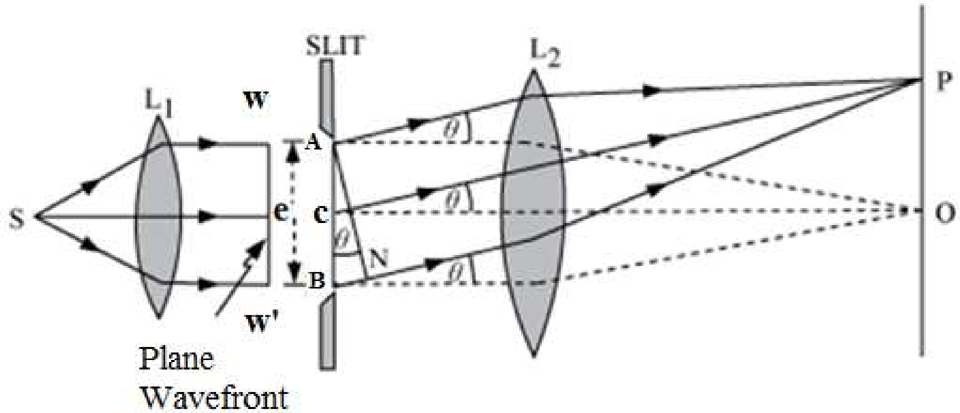
$$90 = \theta_B + r$$

so, angle between reflected and refracted rays is 90°

2. <u>a. Obtain expression for intensity due to Fraunhofer diffraction due at a single slit.</u>

Consider a slit AB of width e perpendicular to the plane of the paper. Let a plane wave front WW' of monochromatic light of wave length λ propagating normally to the slit is incident on AB.

- According to Huygens wave theory every point on the wave front incident on the slit is act as a source of secondary wavefronts.
- These secondary wavefronts travelling normal to the slit along 'CO' are brought to focus at O by using lens L_2 on the screen.
- The secondary wavelets travelling at an angle θ with normal are focused at a point P on the screen.



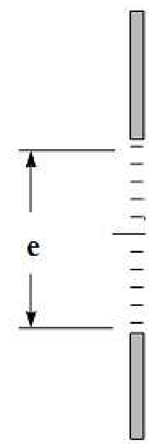
The intensity at these points (O and P) depends on the path difference between secondary waves originating from the slit AB. Since the secondary wavefronts, which are travel normal to the slit have no path difference, therefore, the intensity at point O is maximum and is known as “Central Maximum”.

To find the intensity at P, draw a normal AN on BP. The path difference between extreme rays from slit AB is

$$BN = AB \ \sin\theta \ = e \ \sin\theta$$

$$\therefore \text{ The phase difference } = \frac{2\pi}{\lambda} (e \ \sin\theta)$$

Let the slit AB is divided into large number of n equal parts and the phase difference between any two consecutive parts is equal to



$$\frac{1}{n}\left[\frac{2\pi}{\lambda}(e \ \sin\theta)\right] = d \ (\text{say})$$

The Resultant amplitude at P due to secondary waves from each slit having amplitude **a** and phase difference **d** between successive waves is given by

$$R = a\,\frac{\sin\left(\frac{nd}{2}\right)}{\sin\frac{d}{2}}$$

$$= a\,\frac{\sin\left(\frac{\pi e \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi e \sin}{n\lambda}\right)}$$

$$= a\,\frac{\sin\alpha}{\sin\left(\frac{\alpha}{n}\right)} \qquad \text{where } \alpha = \frac{\pi e \sin\theta}{\lambda}$$

Since n is large when compared with $\frac{\alpha}{n}$, $\sin\left(\frac{\alpha}{n}\right)$ may be replaced by $\frac{\alpha}{n}$

$$R = a\,\frac{\sin\alpha}{\left(\frac{\alpha}{n}\right)} = n\,a\left(\frac{\sin\alpha}{\alpha}\right)$$

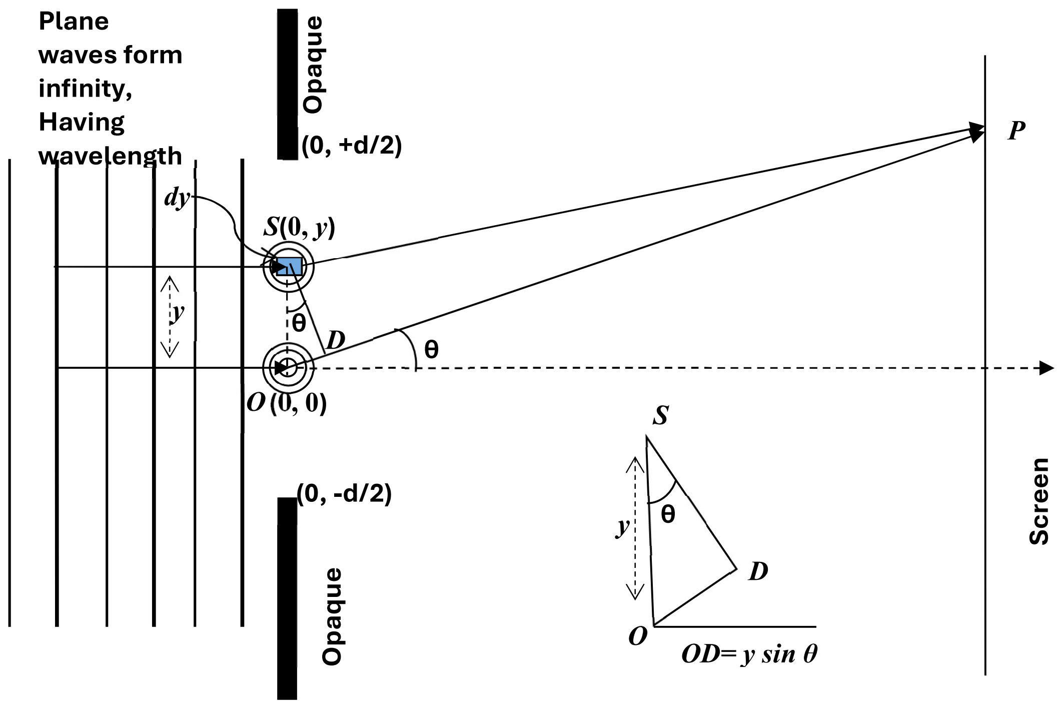
$$= A\left(\frac{\sin\alpha}{\alpha}\right) \quad (\text{where } A = n\,a)$$

$\therefore$ The Resultant intensity at P

$$I = A^2\left(\frac{\sin^2\alpha}{\alpha^2}\right)$$

(OR)

Consider a rectangular slit of width ***d*** extending along y – axis as shown below. Choose the center of this slit as origin with coordinates O (0, 0). The extent of slit is from (0, -d/2) to (0,+d/2).



Let a Plane parallel beam of rays (plane wavefronts) of wavelength λ fall on the slit from left hand side as shown in the figure. Let there be a distant screen with point of observation P making an angle θ with horizontal as shown in the figure. We are interested in calculating the intensity at P due the overlap of all rays (waves) coming from the entire slit.

According to Huygens theory of light every point of the wavefront is a source of secondary spherical wavelets. Let there be a point *S* (0, y) where there is a point spherical source with width *dy* as shown. The wave reaching at P from an arbitrary point *S* (0, *y*) can described as

$$d\psi_P = (a\ dy).e^{i\varphi}$$

Where *a* is the amplitude of received light (plane wave) per unit width of the slit, φ is the phase of the spherical wave reaching at P coming from S. The amplitude of spherical wave at a location of *y* having finite width of *dy* will be *a.dy* .The phase φ can be calculated as

$$\varphi = \frac{2\pi}{\lambda}.path\ (\Delta)$$

Path Δ will be the path difference created between the wave coming directly from the center of the slit at O and the ray coming from S, which is OD from the figure.

$$\varphi = \frac{2\pi}{\lambda}.OD$$

$$\varphi = \frac{2\pi}{\lambda} . y \sin\theta$$

(as OD = *y sinθ* from the Δ ODS)

$$\varphi = qy \quad with\ q = \frac{2\pi}{\lambda} . \sin\theta$$

Finally the *dy* contributes to point P with $\boldsymbol{d\psi_P}$ given by

$$\boldsymbol{d\psi_P = (a\ dy). e^{i\varphi} = a\ dy\ e^{iqy}} \quad \ldots\ldots\ldots\ldots.. (1)$$

Where, *a* is the amplitude per unit width of the slit

dy is the elemental point source located at *y* units distance from origin on the slit

$i = \sqrt{-1}$

$q = \frac{2\pi}{\lambda} . \sin\theta$

θ = angle made by point P at origin

The following figure shows the point spherical sources sending rays that overlap at P on the screen. There are only 8 spherical sources shown, but in reality there will be infinitely many such spherical point sources.

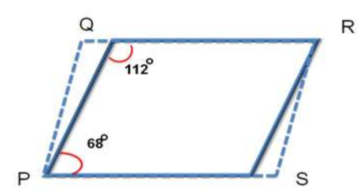
2. b. Describe construction and working of Nicol's prism.

Nicol prism is an optical device which is used for producing and analysing plane polarized light in practice.

Principle

Nicol Prism is based upon phenomenon of total internal reflection and Double refraction. It is constructed in such a way that O-ray is eliminated by total internal reflection and E-ray gets transmitted through it.

Construction

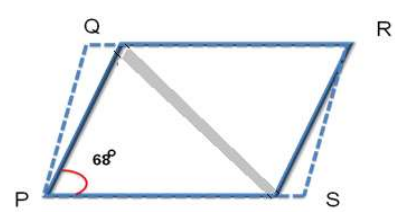


It is constructed from the calcite crystal whose length is three times of its width.

Its end faces PQ and RS are cut such that the angles in the principal section become 68° and 112° .

The crystal is then cut diagonally into two parts.

The surfaces of these parts are grinded to make optically flat and then these are polished.



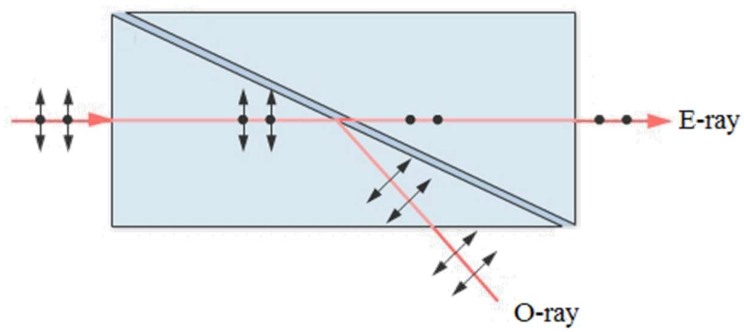
Thus polished surfaces are connected together with a special cement known as Canada Balsam. The refractive index of Canada balsam cement being 1.55 lies between those of ordinary and extraordinary and 1.4864, respectively.

Working:

When a beam of unpolarized light is incident on the face PQ, it gets split into two refracted rays, named O-ray and E-ray. These two rays are plane polarized rays, whose vibrations are at right angles to each other. The Canada Balsam layer acts as an optically rarer medium for the ordinary ray and it acts as an optically denser medium for the extraordinary ray.

• When ordinary ray of light travels in the calcite crystal and enters the Canada balsam cement layer, it passes from denser to rarer medium. Moreover, the angle of incidence is greater than the critical angle, the incident ray is totally internally reflected from the crystal and only extraordinary ray is transmitted through the prism.

• Therefore, fully plane polarized wave is generated with the help of Nicol prism.



3. a. State and explain second law of Thermodynamics.

Second law of thermodynamics:

Second law of thermodynamics is a fundamental law of nature which explains that heat can flow only from hot body to cold body by itself.

There are several statements of this law. Two are the most significant viz.,

(a) **Kelvin- Planck statement:** No process is possible whose sole result is the absorption of heat from a source and the complete conversion of the heat into work. It is impossible to get a continuous supply of work from a body by cooling it to a temperature lower than that of the surroundings.

Ex: Cars and bikes engine: In a car engine and bike engine, there is a higher temperature reservoir where heat is produced and a lower temperature reservoir where the heat is released.

Thus these engines are the example of second law of thermodynamics.

(b) **Clausius statement:** No process is possible whose sole result is the transfer of heat from a colder object to a hotter object. It is impossible for self acting machine to transfer heat from colder body to hotter body without an aid of external agency.

Ex: Refrigerator using electricity to change the direction of heat flow

The heat is traveling from the lower temperature body (i.e inside space of refrigerator) to the higher temperature body (i.e outside the refrigerator).

But this process is not possible on its own. To make this heat flow possible, there is a supply of external energy to this refrigerator. This external energy is nothing but electrical energy which is further used in the compressor of the refrigerator to produce mechanical work.

3. b. Explain entropy and disorder.

Concept of entropy:

The concept of entropy refers to state of order represented by S.

A change in order is a change in the number of ways of arranging the particles, and it is a key factor in determining the direction of any process.

Solid $\longrightarrow$ Liquid $\longrightarrow$ Gas

More Order $\longrightarrow$ Less Order

The increase in entropy is given by dS = $\frac{dQ}{T}$.

Entropy is a state function whose magnitude depends only on the parameters of the system and can be expressed in terms of (P,V,T)

2. dS is a perfect differential. Its value depends only on the initial and final states of the system

3. Absorption of heat increases entropy of the system. In a reversible adiabatic change dq=0, the entropy change is zero

4. For carnot cycle $\oint dS = 0$

5. The net entropy change in a reversible process is zero, $\Delta Suniverse = \Delta Ssystem + \Delta Ssurrounding$ = 0 In irreversible expansion $\Delta Suniv = +ve$ cyclic processs, $\Delta Suniv > 0$.

All-natural process will take place in a direction in which the entropy would increase. A thermodynamically irreversible process is always accompanied by an increase in the entropy of the system and its surroundings taken together while in a thermodynamically reversible process, the entropy of the system and its surroundings taken together remains constant.

When processes occur, in general, they are irreversible and the degree of disorder increases as a result of these processes. As an example, let us take the case of isothermal expansion of an ideal gas. As the gas absorbs heat, it slowly expands. At the end of the process the gas occupies a greater volume than at the beginning. The gas molecules are more disordered now. The gas will not, by its own accord, give up its thermal energy and segregate itself to confine to the initial volume. We, thus, observe that the flow of heat takes place in the direction that increases the amount of disorder. The same type of order to disorder change occurs when free expansion of gas occurs, when one gas diffuses into another, and in similar other spontaneous processes.

4. a. State and explain Carnot's Theorem.

Statement:

No heat engine can have more efficiency than the reversible engine operating between the same source(T1) and sink (T2) and the efficiency of Carnot engine is independent of the nature of the working substance.

Proof: Consider an irreversible engine (I) and a reversible engine(R) working between same source and sink as shown in figure.

Let the engine I extracts Q1 heat from source at T1 and releases Q2 heat to sink at T2 .
The work done by the engine W = Q1 –Q2
Heat given to the sink Q2 =Q1-W -----------------------------(1)

The efficiency η_I= W/Q1 ---(2)

Now the reversible engine R works as refrigerator
This extracts Q2' from sink at T2 and releases Q1 heat to the source at T1 by absorbing work W1 from surroundings
The work done on the system W1 = Q1 – Q2'
Heat absorbed from sink Q2' =Q1-W1 --(3)
The efficiency η_R = W1/Q1

Let the efficiency of I > the efficiency of R

$$\therefore \eta_I > \eta_R \Rightarrow W/Q_1 > W_1/Q_1 \Rightarrow W > W_1$$

By coupling these two engines such that work done by the Irreversible engine can be given to Reversible engine such that the compound engine extracts Q_2' from the sink and releases Q_2 to sink and given by

$$Q_2' - Q_2 = (Q_1 - W_1) - (Q_1 - W) = W - W_1 > 0 \because W > W_1$$

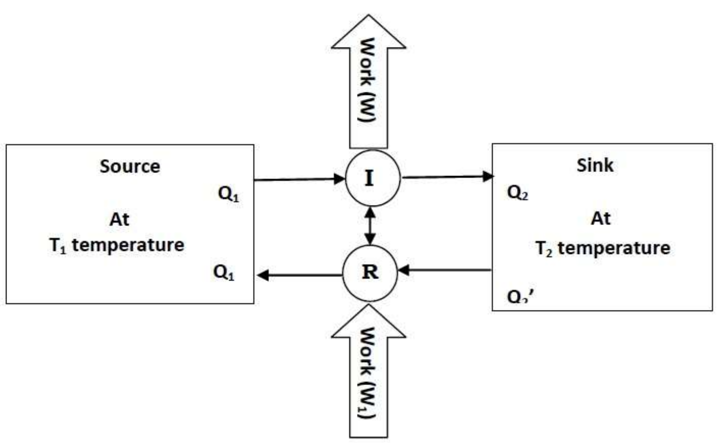
Hence the net heat taken from the sink is positive. But this is impossible for self acting machine extracts heat from cold body without aid of external agency.

∴ Our assumption that the irreversible engine is more efficient than the reversible engine is wrong.

Thus no heat engine working between a Source and sink can be more efficient than a reversible engine.

Since efficiency of engine is given by $\eta = 1 - T_2/T_1$ which is independent of nature of working substance.

Let us consider two reversible engines A and B working between the same source and sink then A cannot be more efficient than B and B cannot be more efficient than A so two heat engines are equally efficient.



4. b. Explain First Law of Thermodynamics:

Statement: If the quantity of heat supplied to a system is capable of doing work, then the quantity of heat absorbed by the system is equal to the sum of the increase in the internal energy and the external work done by it.

$$dQ = dU + dW$$

Where dQ ----The quantity of heat energy supplied to the system.
dU ---- The increase in internal energy.
dW **----** The external work done by the system.

Sign convention:

(1) If the heat energy dQ is added to the system is taken as positive.

(2) If the heat energy dQ is released from the system is taken as negative

(3) The increase in internal energy dU is taken as positive.

(4) The decrease in internal energy dU is taken as negative.

(5) The work is done dW by the system is taken as positive.

(6) The work is done dW on the system is taken as negative.

Significance:

(1) This law verifies the law of conservation of energy in thermodynamics.

(2) This law introduces the concept of internal energy.

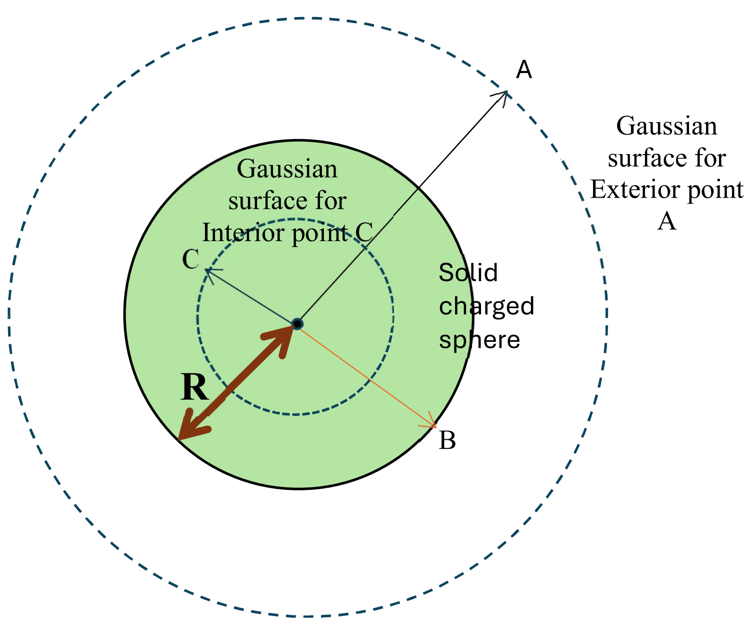
(3) This law is applicable for all the states and natural process.

Limitations:

(1) This law fails to explain the direction of heat flow.

(2) This law fails to explain the concept of entropy.

5. a. Obtain the expression for the electric field due to a solid charged sphere using Gauss law.



Consider a Spherical Charge distribution of Radius R as shown (shaded sphere) above. Let there be a uniform charge of magnitude Q distributed over the sphere with charge density ρ. $\left\{\rho = \frac{Q}{\frac{4}{3}\pi R^3}\right\}$.

We are interested in calculating the electric field due to this charge distribution at three different points near the sphere.

1. Exterior point A: A is at a distance of "r" from the center of this charge distribution (r>R)
2. Surface point B: B is at a distance of R from the center of this charge distribution (r=R)
3. Interior point C: C is at a distance of "r" (r<R) from the center of this charge distribution.

Case1: At A, Let us construct a Gaussian sphere centered at the same center as that for the charge distribution with radius r(>R) shown as a dotted sphere (outer) in the above diagram. Applying Gauss law to this sphere gives,

$$\oiint \vec{E} \cdot \overrightarrow{dS} = \frac{1}{\epsilon_o}\{net\ charge\ enclosed\ by\ the\ Gaussian\ surface\}$$

The total charge enclosed by outer sphere with radius "r" (>R) is Q; $\left\{\rho \frac{4}{3}\pi R^3 = Q\right\}$. As the magnitude of Electric field E on the surface of this sphere with radius "r" remains constant (though its direction will definitely change from point to point on the same sphere), E can be taken outside of the integration. The angle between tiny element $\overrightarrow{dS}$ and $\vec{E}$ is always zero degree as the radius vector of sphere is always parallel to the Electric field at the point on the surface of sphere.

$$\oiint E\ dS \cos 0 = \frac{1}{\epsilon_o}\{Q\} = \frac{1}{\epsilon_o}\left\{\rho \frac{4}{3}\pi R^3\right\}$$

$$E \oiint\ dS\ \times\ 1 = \frac{1}{\epsilon_o}\{Q\} = \frac{1}{\epsilon_o}\left\{\rho \frac{4}{3}\pi R^3\right\}$$

$$\oiint\ dS\ = surface\ area\ of\ sphere\ with\ radius\ (r > R) = 4\pi r^2$$

$$E\ 4\pi r^2\ = \frac{1}{\epsilon_o}\{Q\} = \frac{1}{\epsilon_o}\left\{\rho \frac{4}{3}\pi R^3\right\}$$

$$E(r > R)\ = \frac{Q}{4\pi r^2\epsilon_o} = \frac{1}{4\pi r^2\epsilon_o}\left\{\rho \frac{4}{3}\pi R^3\right\} = \left\{\frac{\rho R^3}{3r^2\epsilon_o}\right\}$$

Case2: At B (on the surface of sphere r = R). Construct another Gaussian sphere like the one in above case with radius r = R with same centre. Apply Gauss law to this sphere.

$$\oiint \vec{E} \cdot \overrightarrow{dS} = \frac{1}{\epsilon_o}\{net\ charge\ enclosed\ by\ the\ Gaussian\ surface\}$$

Using a similar argument like in case 1 the above integral can be written as ,

$$\oiint E\ dS \cos 0 = \frac{1}{\epsilon_o}\{Q\} = \frac{1}{\epsilon_o}\left\{\rho \frac{4}{3}\pi R^3\right\}$$

$$E \oiint\ dS\ \times\ 1 = \frac{1}{\epsilon_o}\{Q\} = \frac{1}{\epsilon_o}\left\{\rho \frac{4}{3}\pi R^3\right\}$$

$$\oiint dS = surface\ area\ of\ sphere\ with\ radius\ (r = R) = 4\pi r^2 = 4\pi R^2$$

$$E\ 4\pi R^2 = \frac{1}{\epsilon_o}\{Q\} = \frac{1}{\epsilon_o}\left\{\rho\frac{4}{3}\pi R^3\right\}$$

$$E(r = R) = \frac{Q}{4\pi R^2\epsilon_o} = \frac{1}{4\pi R^2\epsilon_o}\left\{\rho\frac{4}{3}\pi R^3\right\} = \left\{\frac{\rho R}{3\epsilon_o}\right\}$$

Case3: For interior point at C, a similar application like above holds. Construct a Gaussian sphere with radius r < R from the same center as that of charge distribution. For the application of Gauss law, the total charge contained within this Gaussian sphere needs to be calculated. For the above two cases, the Gaussian surfaces hold same quantity of charge Q, but in this case the total charge enclosed by the smaller sphere is not the whole charge Q but a portion of it.

$$Q_{enclosed} = \rho\frac{4}{3}\pi r^3$$

$$\oiint \vec{E}\cdot\overrightarrow{dS} = \frac{1}{\epsilon_o}\{Q_{enclosed}\} = \frac{1}{\epsilon_o}\left\{\rho\frac{4}{3}\pi r^3\right\}$$

As for this Gaussian sphere too the similar argument holds and the left hand side of the above equation transforms into

$$\oiint E\ dS\cos 0 = E\ 4\pi r^2$$

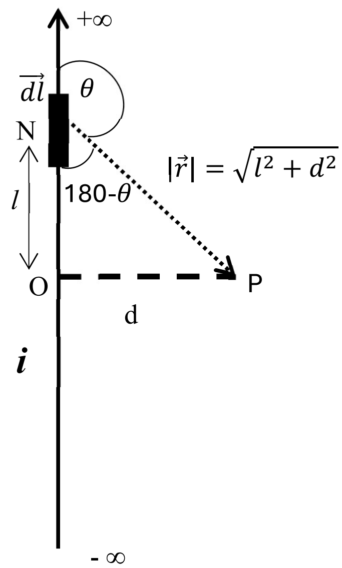
$$E\ 4\pi r^2 = \frac{1}{\epsilon_o}\{Q_{enclosed}\} = \frac{1}{\epsilon_o}\left\{\rho\frac{4}{3}\pi r^3\right\}$$

$$E = \frac{1}{4\pi r^2\epsilon_o}\left\{\rho\frac{4}{3}\pi r^3\right\} = \frac{\rho r}{3\epsilon_o}$$

Using $\rho\frac{4}{3}\pi R^3 = Q$

$$E(r < R) = \frac{Qr}{4\pi R^3\epsilon_o}$$

5.b. Derive the magnetic field due to a straight current carrying conductor using Biot – Savart's law.



Consider a long straight conducting wire carrying a steady current of magnitude "$\boldsymbol{i}$"as shown in the figure above. We are interested in calculating the magnetic field at point P (at distance d from O) due to this current carrying wire.

Consider a small element of current $\vec{dl}$ at point N on the wire which is at distance l from the centre of the wire at O. The magnetic field (dB) at P due to current element at N ($\vec{dl}$) is given by Biot Savart's law,

$$\vec{dB} = \frac{\mu_o}{4\pi} i \frac{\vec{dl} \times \hat{r}}{r^2}$$

Where, r is given by

$$|\vec{r}| = \sqrt{l^2 + d^2}$$

The angle between $\hat{r}$ and $\vec{dl}$ is θ.

$$|\vec{dB}| = \left|\frac{\mu_o}{4\pi} i \frac{\vec{dl} \times \hat{r}}{r^2}\right| = \frac{\mu_o}{4\pi} i \frac{dl\, sin\theta}{r^2}$$

Using ΔOPN,

$$\sin(180 - \theta) = sin\theta = \frac{d}{\sqrt{l^2 + d^2}}$$

Hence,

$$|\vec{dB}| = \frac{\mu_o}{4\pi} i \frac{dl}{r^2} \frac{d}{\sqrt{l^2 + d^2}}$$

Due to entire wire, the magnetic field can be calculated using integration,

$$B = \int |\vec{dB}| = \int_{-\infty}^{+\infty} \frac{\mu_o}{4\pi} i \frac{dl}{r^2} \frac{d}{\sqrt{l^2 + d^2}}$$

$$= \frac{\mu_o}{4\pi} i \int_{-\infty}^{+\infty} \frac{dl}{\left\{\sqrt{l^2 + d^2}\right\}^2} \frac{d}{\sqrt{l^2 + d^2}}$$

$$= \frac{\mu_o}{4\pi} id \int_{-\infty}^{+\infty} \frac{dl}{\{l^2 + d^2\}^{\frac{3}{2}}}$$

Put $l = d\ tan\theta$ implies, $dl = d\ sec^2\theta\ d\theta$

$$upper\ limit:\ +\infty = d\ tan\theta \rightarrow \theta = +\frac{\pi}{2}$$

$$Lower\ limit:\ -\infty = d\ tan\theta \rightarrow \theta = -\frac{\pi}{2}$$

$$B = \frac{\mu_o}{4\pi} id \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{d\ sec^2\theta\ d\theta}{\{(d\ tan\theta)^2 + d^2\}^{\frac{3}{2}}}$$

$$B = \frac{\mu_o}{4\pi} id \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{d\ sec^2\theta\ d\theta}{\{d^2\ sec^2\theta\ \}^{\frac{3}{2}}} = \frac{\mu_o}{4\pi} id \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{d\ sec^2\theta\ d\theta}{(d\ sec\theta)^3} = \frac{\mu_o}{4\pi d} i \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos\theta\ d\theta$$

$$= \frac{\mu_o}{4\pi d} i\{sin\theta\}_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} = \frac{\mu_o}{4\pi d} i(1 - [-1]) = \frac{\mu_o}{2\pi d}$$

Direction of this B is governed by Right hand thumb rule

6.a. Explain Faraday's law of electromagnetic induction

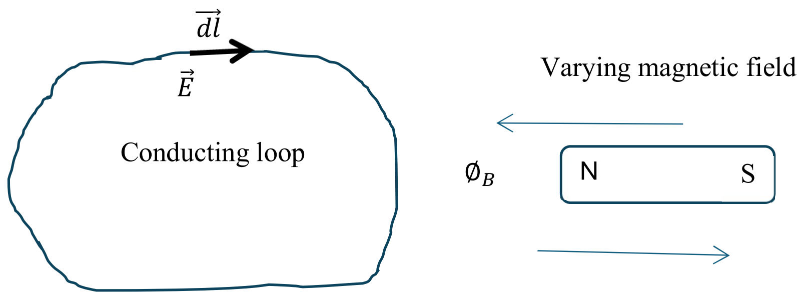
Faraday's Law of electromagnetic induction states the following

Consider a closed conducting loop that is subjected to a time varying magnetic field (somehow the magnetic field changes with time). Faraday's law states that the "Total induced EMF of a closed conducting loop is directly proportional to the time rate of change of magnetic flux associated with that conducting loop".

$$\varepsilon \propto \frac{d\emptyset_B}{dt}$$

Where, ε is the total induced EMF of a conducting loop,

$\emptyset_B$ is the magnetic flux associated with the loop



The proportionality constant in the above equation is chosen as k which is made equal to 1 by defining voltage as 1 volt.

Hence Faraday's law takes a new form

$$\varepsilon = k\frac{d\emptyset_B}{dt} = \frac{d\emptyset_B}{dt}$$

(By choosing K =1 Volt)

From Lenz's law it is further confirmed that the direction of induced EMF is always in such way that in creates an internal magnetic field in the loop, known as induced magnetic field so as to compensate any change that happens in the associated magnetic field of this loop. This induced EMF is always in opposition to the cause that creates this change in magnetic field, hence,

$$\varepsilon = -\frac{d\emptyset_B}{dt}$$

The negative sign indicates the opposition to induced EMF. This total expression is generally called the Faraday's law.

In integral form it can be expressed as

$$\oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d\emptyset_B}{dt}$$

Where $\vec{E}$ is the electric field inside this loop at tiny current element $\overrightarrow{dl}$

In differential form the same equation can be written as

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Where B is the magnetic field at the point of element dl considered above.

6. b. Write Maxwells equations in integral and differential forms.

Integral forms

1. Gauss law in electrostatics:

$$\oiint \vec{E}\cdot\overrightarrow{dS} = \frac{1}{\epsilon_o}\{net\ charge\ enclosed\ by\ the\ \ Gaussian\ surface\ S\}$$

2. Gauss law in Magnetostatics:

$$\oiint \vec{B}\cdot\overrightarrow{dS} = 0$$

The total magnetic flux through any closed surface is zero. Which indicates the absence of magnetic monopoles

3. Faraday's Law of electromagnetic induction:

The induced EMF in a closed conducting loop is always proportional and opposite to the time rate of change of magnetic flux associated with the loop.

$$\varepsilon = -\frac{d\phi_B}{dt}$$

In integral form it can be expressed as

$$\oint \vec{E}\cdot\overrightarrow{dl} = -\frac{d\phi_B}{dt}$$

Where $\vec{E}$ is the electric field inside this loop at tiny current element $\overrightarrow{dl}$

4. Ampere's along with Maxwell's correction: The line integral of magnetic field over a closed loop is always equal to μ_o times the net current (steady current) enclosed by the loop. This law is applicable only to steady currents. Later Maxwell has modified it to the following form

$$\oint \vec{B}\cdot\overrightarrow{dl} = \mu_o\left\{J_{steady} + \epsilon_o\frac{\partial\vec{E}}{\partial t}\right\}\ times\ area\ of\ the\ loop$$

The term, $\epsilon_o\frac{\partial\vec{E}}{\partial t}$ is called the displacement current which was added to Ampere's law by James's clerk Maxwell.

Differential forms for the same equations mentioned above are as follows:

1 $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_o}$

2 $\vec{\nabla}\cdot\vec{B} = 0$

3 $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

4 $\vec{\nabla}\times\vec{B} = \mu_o\left\{J_{steady} + \epsilon_o\frac{\partial\vec{E}}{\partial t}\right\}$

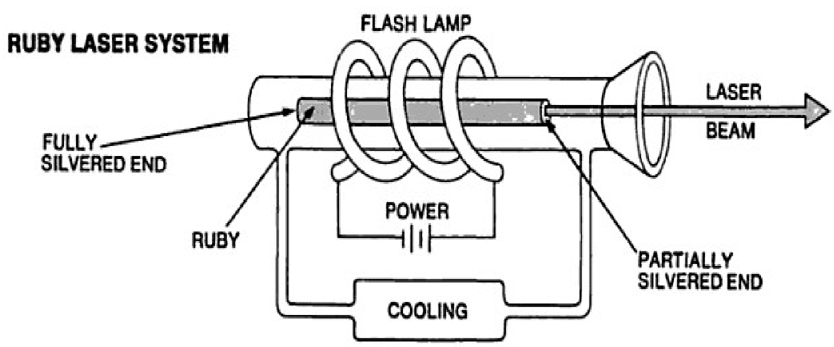
7. a. Describe the construction and working of Ruby Laser.

Ruby Laser

- It is the first solid state three level laser invented by T H Maiman in the year 1960.
- It produces a high output power in the order of Mega Watts with ten nanoseconds pulse duration.
- Pumping Method: Optical Pumping Scheme.
- Laser Active Medium : Chromium atoms

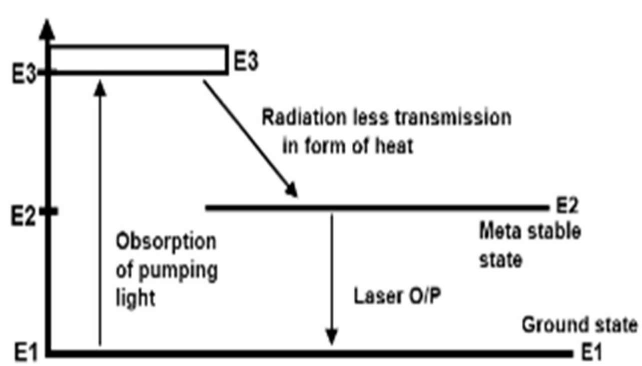
Construction

1. It consists of a long cylindrical ruby rod which is made up of Aluminum Oxide (Al_2O_3) and is doped with 0.05% of Cr_2O_3.
2. Due to the presence of Cr^{3+} ions the ruby rod is appears as pink in colour. This suggest that there is a strong absorption in the visible region.
3. The length of the ruby rod is about 10cm and diameter is 0.5cm.
4. The end faces of the rod are grounded and polished such that the end faces are exactly parallel to each other.
5. One of the ends is silvered for 100% reflection and the another end is silvered for nearly 90% reflection.
6. A helical Xenon flash lamp is surround the ruby rod and is connected to a high voltage trigger pulse (~20kV).
7. During the process, a large amount of heat is produced. So, the system is cooled with the help of a coolant (water) circulating around the ruby rod.



Working

1. Laser action will be takes place between the energy levels of the Cr^{3+} ions. Al_2O_3 is an insulator and the introduction of Cr^{3+} ions results in additional energy levels within the band gap.
2. The Chromium ions having three active energy levels known as E1 - Ground state, E2- Meta stable state and E3- Higher excited state.



3. E3 state is fairly wide and hence can accept a wide range of wavelengths. It has short life time.

4. In this the lasing action occurs between E2 and E1.
5. When ruby rod is irradiated with flash lamp, Chromium ions absorb the light of wavelength at around 5600A° (5000-6000A°) which will be either green or blue color.
6. As a result the ions transferred to higher excited state E3 from Ground state E1.
7. From this level (E3) the ions will go down to Meta stable state (E2) in a non-radiative transition. This energy is transferred to the crystal vibrations and changed into heat.
8. Since the life time of the E2 level is in the order of milliseconds, Chromium ions remain in this level for longer duration.
9. So population inversion takes place between Meta stable state (E2) and Ground State (E1).
10. The spontaneously emitted initial photons would travel in all the directions, of these, those travelling parallel to the axis of the rod would be reflected at the ends and pass many times through the amplifying medium and stimulate the atoms in Meta Stable state.
11. As a result stimulated emission takes place and chromium ions translate from E2 to E1.
12. This transition gives rise to the emission of light of wavelength 6943A°.
13. The output of this laser consists of a series of laser pulses for duration of milliseconds or less and the diameter of the beam is 1 mm to 25mm.

7. **b. Explain different types of losses in optical Fibers**

Losses in Optical fibres

When light propagates through an optical fibre, a small percentage of light is lost through different mechanisms. The loss of optical power is measured in terms of decibels per kilometre
for attenuation losses.
Attenuation: It is defined as the ratio of the optical power output (Pout) from a fibre of length 'L' to the power input (Pin). $Attenuation\ (\alpha) = (-10/L) \log Pout/Pin \quad dB/K$m.

Scattering:
Scattering is also a wavelength dependent loss, which occurs inside the fibers. Since the glass is used in fabrication of fibers, the disordered structure of glass will make some variations in the refractive index inside the fiber. As a result, if light is passed through the atoms in the fiber, a portion of light is scattered (elastic scattering). This type of scattering is called Raleigh scattering.

$$\text{Raleigh scattering loss} \propto \frac{1}{\lambda^4}$$

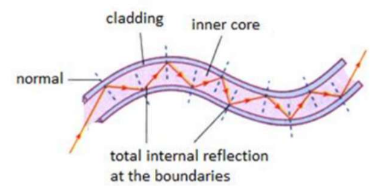
Radiative loss occurs in fibres, due to bending of finite radius of curvature in optical fibres. The types of bends are (a) Macroscopic bend and (b) Microscopic bend

Macroscopic bends: If the radius of core is large compared to fibre diameter as shown in figure, it may cause large- curvature at the position where the fibre cable turns at the corner. At these corners the light will not satisfy the condition for total internal reflection and hence it escapes out from the fibre. This is called as macroscopic/macro bending losses. Also note that this loss is negligible for small bends.

Microscopic bends: Micro-bends losses are caused due to non-uniformities or micro-bends inside the fibre as shown in figure. This micro bends in fibre appears due to non-uniform pressures created during the cabling of the fibre or even during the manufacturing itself. It leads to loss of light by leakage through the fibre.

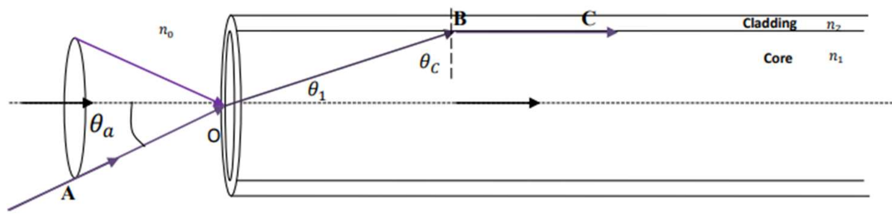
8. a. Explain the propagation of light through an optical fibre and obtain the expression its numerical aperture.

The light entering through one end of core strikes the interface of the core and cladding with angle greater than the critical angle and undergoes total internal reflection. After series of such total internal reflection, it emerges out of the core. Thus, the optical fiber works as a waveguide. Care must be taken to avoid very sharp bends in the fiber because at sharp bends, the light ray fails to undergo total internal reflection.



Expression for Numerical aperture and Condition for propagation:

Consider a ray of light in a medium of RI 'n_0' entering in to a fiber having a core of RI 'n_1' and cladding of RI 'n_2' at a point "O" on the core surface. The ray OA incident at O, at an angle θ_a refracts in to the core at an angle θ_1 and falls on the core-cladding interface at an angle θ_C at B and grazes the interface along BC after refraction.



For the refraction at 'O', Snell's law can be written as $\boldsymbol{n_0 \sin\theta_a = n_1 \sin\theta_1}$

$$\therefore \sin\theta_a = \left(\frac{n_1}{n_0}\right)\sin\theta_1 \quad ....(1)$$

Similarly For the refraction at 'B', Snell's law becomes $\boldsymbol{n_1 \sin\theta_C = n_2 \sin 90°}$

But we have $\theta_C = (90° - \theta_1)$ $\quad \therefore \; n_1 \sin(90° - \theta_1) = n_2$

$$\text{Or} \quad \cos\theta_1 = \left(\frac{n_2}{n_1}\right)$$

$$\Rightarrow \quad \sin\theta_1 = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} = = \sqrt{\frac{(n_1^2 - n_2^2)}{n_1^2}} \quad .... (2)$$

$$\therefore \text{Eqn. (1) becomes} \quad \sin\theta_a = \left(\frac{n_1}{n_0}\right) \cdot \sqrt{\frac{(n_1^2 - n_2^2)}{n_1^2}}$$

$$\Rightarrow \quad \text{N. A.} = \sin\theta_a = \frac{\sqrt{(n_1^2 - n_2^2)}}{n_0}$$ (This is the expression for the Numerical aperture.)

If the surrounding medium is air then N . A . $=\sin\theta_a = \sqrt{(n_1^2 - n_2^2)}$ .... (3)

8. b. Distinguish between spontaneous emission and stimulated emissions.

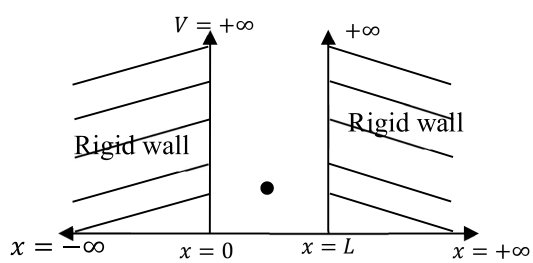
S.no	Stimulated Emission	Spontaneous emission
1.	An atom in the excited state is induced to return to the ground state , thereby resulting in two photons of same frequency and energy is called Stimulated emission	The atom in the excited state returns to the ground state thereby emitting a photon, without any external inducement is called Spontaneous emission.
2.	The emitted photons move in the same direction and is highly directional	The emitted photons move in all directions and are random
3.	The radiation is highly intense , monochromatic and coherent	The radiation is less intense and is incoherent.
4.	The photons are in phase, there is a constant phase difference.	The photons are not in phase (i.e.) there is no phase relationship between them.
5.	The rate of transition is given by $R_{21}(St) = B_{21}\rho_v N_2$	The rate of transition is given by $R_{21}(SP) = A_{21}N_2$

9.a. Apply Schrodinger's equation to a particle in a one-dimensional box and obtain the energy values and wave function

Consider a particle with mass ***m*** and total energy ***E*** confined to a deep one dimensional potential well as shown in the figure below.

When compared with the height (infinite) of the walls at x = 0 and at x = L, the potential energy of the particle is negligible, say zero. V = 0, The walls at x = 0 and at x = L are perfectly rigid extending up to positive and negative infinity on the x – axis.

We can apply the



Schrödinger time independent wave equation in the region, x = 0 and at x = L;

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

As V = 0,

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2mE}{\hbar^2}\psi = 0$$

Put,

$$\frac{2mE}{\hbar^2} = k^2$$

$$\frac{\partial^2\psi}{\partial x^2} + k^2\psi = 0$$

General solution to this differential equation will be

$$\psi(x) = A\exp(ikx) + B\exp(-ikx)$$

The complete solution can be obtained by using boundary conditions on ψ at x = 0 and at x = L.

BOUNDARY CONDITION:

ψ Must vanish at x = 0 and at x = L as the particle does not enter the infinite potential energy zone.

At x = 0;

$$\psi(x) = 0 = A\exp(ik \cdot 0) + B\exp(-ik \cdot 0)$$

$$0 = A + B$$

Or

$$B = -A$$
$$\therefore\ \psi(x) = A\exp(ikx) - A\exp(-ikx)$$
$$\psi(x) = A(\exp(ikx) - \exp(-ikx))$$
Multiply and divide by $2i$,
$$\psi(x) = 2Ai\left(\frac{\exp(ikx) - \exp(-ikx)}{2i}\right)$$
$$\left(\frac{\exp(ikx) - \exp(-ikx)}{2i}\right) = sin\, kx$$
$$\psi(x) = 2Ai\, sin\, kx$$
Similarly at x = L;
$$\psi(x = L) = 0 = 2Ai\, sin\, kL$$
As A is not zero, for non – trivial solution,
$$sin\, kL = 0 = \sin n\pi$$
$$k = \frac{n\pi}{L}$$
With n taking all positive and negative integer values including zero.
But physically, n = 0 solution does not make any sense. If this is true then k will go to zero and hence E will go to zero, a particle without energy. This is really not of interest.
$$\psi(x) = 2Ai\, sin\frac{n\pi}{L}x$$
n = ±1, ±2, ±3…
But still this wave function is still not complete because A is still unknown. Let us fix the value of this A by using Max Born's probability normalization condition,
$$\int_{-\infty}^{\infty} \psi(x)^*\, \psi(x)\, dx = 1$$
As the particle does not enter the regions x = - ∞ to x = 0 and x = 0 to x = + ∞, the wave function becomes zero in these regions to make the probability completely zero in these regions.
$$\int_{-\infty}^{0} \psi(x)^*\, \psi(x)\, dx + \int_{0}^{\mathrm{L}} \psi(x)^*\, \psi(x)\, dx + \int_{\mathrm{L}}^{\infty} \psi(x)^*\, \psi(x)\, dx = 1$$
The first and Third parts of the above integral evaluates to zero and hence,
$$\int_{0}^{\mathrm{L}} \psi(x)^*\, \psi(x)\, dx = 1$$
Where,
$$\psi(x)^* = 2A^* i^*\, sin\frac{n\pi}{L}x$$
Is the complex conjugate of the wave function. It can be obtained by replacing the i 's with their negatives.
$$i^* = -i$$
$$\int_{0}^{\mathrm{L}} \left(2A^*(-i)\, sin\frac{n\pi}{L}x\right)\left(2Ai\, sin\frac{n\pi}{L}x\right)\, dx = 1$$
$$4\int_{0}^{\mathrm{L}} (A^*A)\left(sin\frac{n\pi}{L}x\right)^2\, dx = 1$$
$$4\int_{0}^{\mathrm{L}} |A|^2\left(\frac{1 - cos\, 2\frac{n\pi}{L}x}{2}\right) dx = 1$$

$$|A|^2 \int_0^L \left(\frac{1 - cos\, 2\frac{n\pi}{L}x}{2} \right) dx = \frac{1}{4}$$

$$\int_0^L \left(\frac{1 - cos\, 2\frac{n\pi}{L}x}{2} \right) dx = \left(\frac{x - \frac{L}{2n\pi} sin\, 2\frac{n\pi}{L}x}{2} \right)_0^L$$

$$= \frac{L}{2}$$

$$\therefore \quad |A|^2 \int_0^L \left(\frac{1 - cos\, 2\frac{n\pi}{L}x}{2} \right) dx = \frac{1}{4}$$

$$|A|^2 \frac{L}{2} = \frac{1}{4}$$

$$\therefore A = \frac{1}{\sqrt{2L}}$$

$$\therefore \psi(x) = 2\frac{1}{\sqrt{2L}} i\, sin \frac{n\pi}{L} x$$

Eigen functions of particle in one dimensional box are given by

$$\boxed{\psi_n(x) = \pm i \sqrt{\frac{2}{L}} sin \frac{n\pi}{L} x \qquad n = \pm 1, \pm 2, \pm 3 \ldots}$$

Energy of the particle can be calculated by using

$$E\, \psi \equiv \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\, \psi$$

As the particle has no potential energy, V = 0;

$$E\, \psi_n(x) = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \left(i \sqrt{\frac{2}{L}} sin \frac{n\pi}{L} x \right)$$

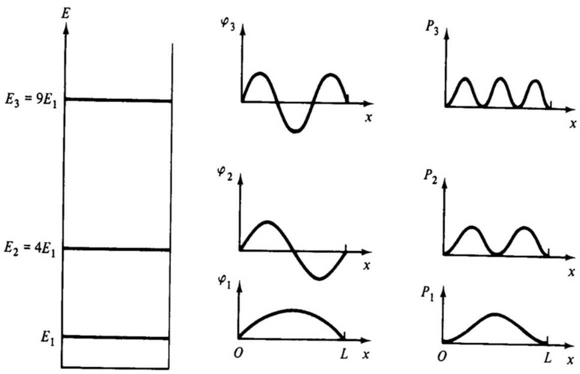
$$E\, \psi_n(x) = \left(\frac{n\pi}{L} \right)^2 \frac{\hbar^2}{2m} \left(i \sqrt{\frac{2}{L}} sin \frac{n\pi}{L} x \right)$$

$$E\, \psi_n(x) = \left(\frac{n\pi}{L} \right)^2 \frac{\hbar^2}{2m} \psi_n(x)$$

Hence the energies of the particle in a box are quantized according to the above expression.

$$\boxed{E_n = \frac{n^2 \hbar^2 \pi^2}{2mL^2} \qquad n = \pm 1, \pm 2, \pm 3 \ldots}$$

The following graphs respectively represent the energies, wave functions and probabilities of particle in various states, i.e. ground state and excited states.



9. b. Distinguish between qubits and classical bits

S.No.	Classical Bits	Quantum Bits
1.	A Bit, also called Binary Digit or Classical Bit, is the smallest unit of information measurement in digital computing technology.	A Quantum Bit, also called Qubit, is the smallest unit of information measurement in quantum computing.
2.	A bit can have only two values, i.e. 0 and 1.	A quantum bit can have multiple values simultaneously.
3.	Classical bit does not follow superposition property.	Quantum bit follows superposition property.
4.	Bits are inherently stable, i.e. they do not change their states in the absence of external force.	Quantum bits are inherently unstable, i.e. they can change their states even no external force exists.
5.	The value or state of a bit can be determined precisely. Hence, they are deterministic.	The value or state of a quantum bit cannot be precisely determined. Hence, they are probabilistic.
6.	Bits are physically implemented through electronic and optical devices.	Quantum bits are implemented by using quantum systems like ions, atoms, superconductors, etc.
7.	Boolean operations are executed on bits.	Quantum operations are executed on quantum bits.
8.	Bits can be copied perfectly.	Quantum bits cannot be copied perfectly.
9.	The operations on bits are performed using digital logic gates, such as AND, OR, NOT, etc.	The operations on quantum bits are performed using quantum logic gates.

10.a. Obtain the expression for the wavelength of matter waves (de-Broglie's relation) and explain physical significance of wave function.

de-Broglie wavelength is the wavelength associated with a particle in motion, based on its momentum and mass

de-Broglie derived his equation using well established theories through the following series of substitutions:
de Broglie first used Einstein's famous equation relating matter and energy:

$$E=mc^2$$

with

- E = energy,
- m = mass,
- c = speed of light

Using Planck's theory which states every quantum of a wave has a discrete amount of energy given by Planck's equation:

$$E=h\nu$$

with

- E = energy,
- h = Plank's constant (6.62607×10^{-34} J s),
- ν= frequency

Since de-Broglie believed particles and wave have the same traits, he hypothesized that the two energies would be equal:

$$mc^2 = h\nu$$

Because real particles do not travel at the speed of light, de-Broglie submitted velocity (v) for the speed of light (c).

$$mv^2 = h\nu$$

$$\lambda=hv/mv^2=h/mv$$

The complex mathematical function in time and space that describes the de Broglie matter wave associated with moving particles is named a Wave function.

$$\psi = \psi_o \sin{(kx \pm \omega t)}$$

Where, ψ_o is a complex mathematical function Properties of Wave Function

There must be a single value for **Ψ**, and it must be continuous.

It is easy to compute the energy using the Schrodinger equation.

Wave function equation is used to establish probability distribution in 3D space.

If there is a particle, then the probability of finding it becomes 1.

10. b. Explain the basic idea of quantum teleportation.

Quantum teleportation:
Quantum teleportation is a way to transfer the state of a quantum particle (like a photon or an electron) from one place to another without physically moving the particle itself. Let's break it down step-by-step using the example of Alice and Bob:

Step1: Sharing a Quantum Link
Alice and Bob start by sharing a special pair of quantum particles, like photons, that are entangled. Entanglement means the particles are linked in a way such that the state of one particle instantly relates to the state of the other, no matter how far apart they are.Think of it like Alice and Bob each having one half of a magical pair of gloves: if Alice has the left-hand glove, Bob automatically has the right-hand glove.

Step2: Alice Wants to Send a Quantum State
Alice has another quantum particle (let’s call it "Charlie") whose state she wants to send to Bob. The state is something we can’t directly measure or copy because of the rules of quantum physics.

Step3: Alice Performs a Special Measurement
Alice takes her particle (Charlie) and her half of the entangled pair. She performs a special measurement on these two particles, which does two things:

A) It destroys the state of her original particle (Charlie).
B) It creates a set of classical measurement results (two bits of information) that tell Bob what adjustments he needs to make to his particle.

Step 4: Alice Sends Bob the Information
Alice sends the two bits of classical information (via email, phone, or any normal method of communication) to Bob. These bits don’t carry the quantum state itself but are instructions for Bob.

Step5: Bob Recreates the State
Using the classical information from Alice and his half of the entangled pair, Bob applies the right adjustments to his particle. This "transforms" his particle into the exact same state that Alice’s original particle (Charlie) was in.

The Cool Part in this method is Alice’s original particle is destroyed during the process, so there’s no duplication (this is due to quantum rules, measurement always disturbs the systems).
The quantum state is transferred, not the particle itself. It’s as if Bob now has Charlie’s "soul" in his particle. This is brief idea of explaining the quantum teleportation happening between two entities. Quantum teleportation is a key idea in building quantum networks and could one day help create super-secure communication systems or even quantum computers that work across long distances.